A modified Apriori algorithm to generate rules for inference system with probabilistic-fuzzy knowledge base

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Abstract: In the article the possibility of applying one of the methods of data mining – fuzzy association rules to model fuzzy systems is presented. The algorithm of automatic extracting knowledge base is a modification of the Apriori algorithm. The obtained rules of the model contain weights, that are calculated on the basis of measures of fuzzy association rules and constitute probability of fuzzy events contained in the rules. The rules' weights are employed during system inference. The method has been tested using exemplary data.

Keywords: Fuzzy modelling, Knowledge-based systems, Rule-based systems, Probabilistic-fuzzy knowledge base, Identification algorithms, Algorithm Apriori, Fuzzy association rules, Data mining.

1. INTRODUCTION

The *principle of incompatibility*, formulated by L.A. Zadeh says: …'as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics' (Zadeh, 1973). All attempts to bring a lot of details to very complicated issues lessen the reliability of results (Łęski, 2008). Due to employing imprecise terms of natural language in fuzzy systems, one may attempt to reflect even the most complex reality.

The articles (Walaszek-Babiszewska, 2006, 2007) (Błaszczyk, 2008) broached the topic of inference system with probabilistic-fuzzy knowledge base, where linguistic knowledge is contained in weighted IF-THEN rules. The weights constitute marginal and conditional probabilities of fuzzy events in antecedent and consequent. A fuzzy system is assumed to allow a simplified reconstruction of a complicated research problem. However, having many system variables and a large number of fuzzy sets, it takes a lot of complex calculations to identify such model. Moreover, considering the total probability distribution of fuzzy events, the number of elementary rules of the knowledge base highly increases with the number of model variables and the number of fuzzy sets of variables. A large number of rules affects not only identification time of the model, but also causes difficulties with inference with the use of the created knowledge base.

In the articles (Błaszczyk, 2007), (Walaszek-Babiszewska, Błaszczyk, Czabak, 2008) employing one of the methods of data mining was suggested, the method of searching association rules to build a probabilistic-fuzzy rule base. Preliminary granulation of the space of variables in order to find reliable rules with a probability distribution of non-fuzzy events and further fuzzification of events lessen complexity of the model in terms of the number of rules and slightly reduce complexity of calculations. The possibility of employing a modified Apriori algorithm to generate rules for the inference system with a probabilistic-fuzzy knowledge base will be presented in the article. The algorithm will allow to directly find reliable fuzzy rules with probabilities of fuzzy events from the rules' antecedents and consequents.

2. INFERENCE SYSTEM WITH PROBABILISTIC-FUZZY KNOWLEDGE BASE

The structure of the inference system with probabilistic-fuzzy knowledge base is depicted in Fig. 1.

The system consists of the following parts:

- knowledge base, which consists of the knowledge representation for a particular problem,
- fuzzification block, which changes input signals from the field of quantity to quality measures represented by fuzzy sets on the basis of grades of membership,
- inference system, which uses knowledge base and implemented methods in order to solve specialized problems,
- defuzzification block, which calculates output crisp value on the basis of the output membership function.

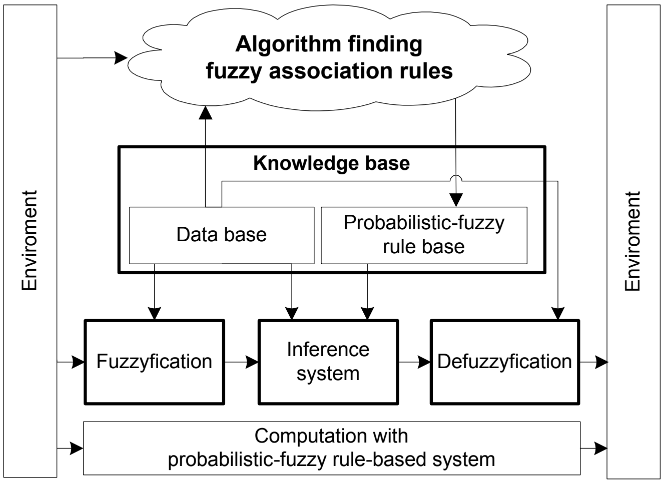

Fig.1. Structure of inference system with probabilistic-fuzzy knowledge base.

There are two basic components of the probabilistic-fuzzy knowledge base: data base and rule base (Herrera, 2005).

A characteristic feature of the rule base of the MISO system is presentation of knowledge in the form of file rules as follows (Walaszek-Babiszewska, 2007) (Yager, Filev, 1995):

$$\begin{aligned} (w_j)\quad & \text{IF } x_1 \text{ is } A_j^1 \text{ AND ..AND } x_N \text{ is } A_j^N \\ & \text{THEN } y \text{ is } B_{1/j} \quad (w_{1/j}) \\ & \text{ALSO } y \text{ is } B_{2/j} \quad (w_{2/j}) \\ & \quad\ldots \\ & \text{ALSO } y \text{ is } B_{M/j} \quad (w_{M/j}) \end{aligned} \tag{1}$$

where:

N – number of input variables of the model,

$x_1, \ldots, x_N$ – input variables of the model, $x_n \in X_n \subset \mathrm{R}$, $n=1,\ldots,N$,

y – output variable of the model, $y \in Y \subset \mathrm{R}$,

A_j^n – j-th linguistic value of n-th input variable, $j=1,\ldots,J$, $n=1,\ldots,N$,

$B_{m/j}$ – linguistic value of output variable in m-th elementary rule of j-th file rule, $m=1,\ldots,M$, $j=1,\ldots,J$,

w_j – weight of j-th file rule constituting marginal probability of simultaneous fuzzy events in the rule antecedent – $P((x_1 \text{ is } A_j^1) \cap \ldots \cap (x_N \text{ is } A_j^N))$. (2)

$w_{m/j}$ – weight of the elementary rule consequent constituting conditional probability of fuzzy events:

$$P((y \; is \; B_{m/j})/((x_1 \; is \; A_j^1) \cap \ldots \cap (x_N \; is \; A_j^N))) = \frac{P((x_1 \; is \; A_j^1) \cap \ldots \cap (x_N \; is \; A_j^N) \cap (y \; is \; B_{m/j}))}{P((x_1 \; is \; A_j^1) \cap \ldots \cap (x_N \; is \; A_j^N))}. \tag{3}$$

Data base contains linguistic values $\{ A_j^n, B_{m/j}, j=1,\ldots,J, n=1,\ldots,N, m=1,\ldots,M\}$ of variables considered in linguistic rules and definitions of fuzzy sets identified with these values. There are mostly quantitative data in the model so spaces of variables are discretized on K disjoint intervals of variable values, correspondingly $a_n = (a_n^1, \ldots, a_n^k, \ldots, a_n^K)$ for input variable x_n and $b = (b^1, \ldots, b^k, \ldots, b^K)$ for output variable y. Number K is selected according the number of measurements and how detailed calculation we need. Definitions of fuzzy sets in case of disjoint intervals of variable values are defined as grades of membership of the intervals to fuzzy sets A_j^n and $B_{m/j}$, and they are marked as $\mu_{A_j^n}(a_n^k) \in [0,1]$ for n-th input variable x_n and $\mu_{B_{m/j}}(b^k) \in [0,1]$ for output variable y. Grades of membership are defined with the following dependency:

$$\sum_{j=1}^{J} \mu_{A_j^n}(a_n^k) = 1, \quad k = 1, \ldots, K \tag{4}$$

for every input variable x_n, $n=1,\ldots,N$ and

$$\sum_{m=1}^{M} \mu_{B_{m/j}}(b^k) = 1, \quad k = 1, \ldots, K \tag{5}$$

for output variable y.

More information on probabilistic-fuzzy representation of systems can be found in articles (Walaszek-Babiszewska, 2006, 2007) (Walaszek-Babiszewska, Błaszczyk, Czabak, 2008).

3. DATA MINING AS A METHOD OF AUTOMATIC EXTRACTING OF KNOWLEDGE BASES

The gist of fuzzy systems is the possibility of using an expert's knowledge on the described phenomena. However, the method of extracting knowledge is efficient only with research problems an expert is familiar with. As there are more and more facts and the information technology connected to collecting and managing large quantities of data has developed, there appeared a need to develop the field of studies concerning automatic extracting knowledge bases. Then the classic methods, as Wang-Mendel method (1992), Nozaki-Ishibuchi-Tanaki method (1997) were developed and also methods belonging to the area of data mining were applied.

Data mining, as the main stage of the process of knowledge discovery (Fayyad, Piatetsky-Shapiro, Smyth, 1996) focuses on non-trivial algorithms of discovering "hidden", so far unknown and potentially important information in data (Frawley, Piatetsky-Shapiro, Matheus, 1992) and saving it as patterns and models.

Some of data mining methods identify regions in space of system variables, that later constitute fuzzy events in rules. This may be achieved by creating clusters with the use of clustering algorithms or identification with the use of the so called covering (separate and conquer) algorithm. Other methods base on constant fuzzy division for each attribute (the so called "fuzzy grid") and each element of the grid is considered as a potential component of the rule. In the first approach each identified rule has its own fuzzy sets. Therefore, the second approach is more favourable from the standpoint interpretation of rule (Hüllermeier, 2005).

Rules (like Mamdani-Assilan) are created by "maximum matching" which may be understood as finding maximally supporting rules. In such case, rules as (1) may be analyzed as coexistence of fuzzy values in collections of empirical data, which constitutes the sense of fuzzy association rules. Weights of rules may be calculated with the use of "fuzzy grid" on the basis of main measures of quality association rules (subchapter 4.1.).

Other of methods of data mining are also hybrid methods, a combination of methodology of fuzzy sets and neural networks (genetic algorithms). Combination of data exploration by the method of fuzzy association rules with the use of genetic algorithms may be found in (Alcalá-Fdez, Alcalá, Gacto, Herrera, 2009).

4. GENERATING RULES BY MEANS OF MODIFIED APRIORI ALGORITHM

4.1. Mining fuzzy association rules

Formally, fuzzy association rules have the form of implication (Kuok, Fu, Wong, 1998):

$$\text{IF } X \text{ is } A \text{ THEN } Y \text{ is } B\ (w, c) \tag{6}$$

where $X=\{x_1,...,x_p\}$ and $Y=\{y_1,...,y_q\}$ represent variables (attributes) – a set of items (also called itemsets) and $A=\{A_{x_1},\ldots,A_{x_p}\}$ and $B=\{B_{y_1},\ldots,B_{y_q}\}$ contain fuzzy sets for corresponding variables X and Y.

Each association rule is connected with two statistical measures defining importance and strength of a rule: support (marked w, in data mining terminology often referred to as *Sup*) – probability of simultaneous occurrence of the set $A_{x_1}\cap\ldots\cap A_{x_p}\cap B_{y_1}\cap\ldots\cap B_{y_q}$ in a collection of sets and confidence (marked c, in data mining terminology often referred to as *Conf*) – conditional probability $P(B_{y_1}\cap\ldots\cap B_{y_q} \mid A_{x_1}\cap\ldots\cap A_{x_p})$. Frequent set is a term used to describe a set of events for which the value of support is larger or equal to an estimated minimum support value.

Iterative Apriori algorithm is considered to be a basic algorithm of discovering association rules (Agrawal, Imielinski, Swami, 1993). This algorithm has a lot of modifications, that are supposed to improve its efficiency (e.g. AprioriTid, AprioriHybrid), but also allow to discover fuzzy association rules (Kuok, Fu, Wong, 1998) (Hong, Kuo, Chi, 2001) (Chen, Wei, 2002). There exists also another more computationally efficient algorithm called FP-Growth (Han, Pei, Yin, 2000), which discovers association rules without candidate itemset generation, but this algorithm supports calculating only "no-fuzzy rules".

The article depicts its own version of a modified Apriori algorithm in order to use it to generate a knowledge base with probabilistic-fuzzy rules.

4.2. Algorithm of the proposed method in detail

Input assumptions of the proposed algorithm are a pre-defined database and minimum support threshold (*min w*). The following is the notation used to present the algorithm:

I – number of measurements used in identification of the model,

N+1 – total number of variables (N input variables, 1 output variable),

K – the number of disjoint intervals of equal width in variable spaces,

x_n – input variables of the model, $x_n \in X_n \subset \mathrm{R}$, n=1,...,N,

y – model output variable, $y \in Y \subset \mathrm{R}$,

$|A_n|$ ($|B|$) – number of linguistic values for n-th input variable (output variable),

A_j^n – j-th linguistic value of n-th input variable, j=1,…,$|A_n|$, n=1,...,N,

B_j – j-th linguistic value of output variable, j=1,…,$|B|$,

$\mathbf{a}_n=(a_n^1,\ldots,a_n^k,\ldots,a_n^K)$ – disjoint intervals of n-th input variable x_n, n=1,...,N,

$\mathbf{b}=(b^1,\ldots,b^k,\ldots,b^K)$ – disjoint intervals of output variable y,

w – calculated support value for itemsets candidates,

min w – estimated minimum support value,

C_r – a set of itemsets candidates that consists of sets of linguistic values for r ($1 \le r \le N$+1) system variables,

F_r – a set of frequent itemsets that consists of sets of linguistic values for r ($1 \le r \le N$+1) system variables,

$A_{j(r)}^{n(r)}$ – $j(r)$-th linguistic value of $n(r)$-th input variable, that is the r-th element in itemsets,

D – empirical data concerning the examined system, in data mining terminology often referred to as transaction data.

D^i – i-th set of empirical values of the model variables $\{x_1^i,\ldots,x_N^i,y^i\}$, i=1,...,I.

The algorithm to generate rules for inference system of MISO type with probabilistic-fuzzy knowledge base is presented below.

Step 1: Discretization of numbers of each set D^i (i=1,...,I) on K disjoint intervals ($\boldsymbol{a}_n$; n=1,...,N and $\boldsymbol{b}$) of equal width in variable space.

Step 2: On the basis of D, calculation of probability distribution for a non-fuzzy event

$$p_k(x_1,..x_n,y)=P(x_1\in a_1^k,\ldots,x_n\in a_n^k,y\in b^k) \tag{7}$$

k=1,...,K, n=1,...,N.

Step 3: Defining candidates of one-element frequent sets C_1. Calculation of support for input variables

$$w(A_j^n) = P(A_j^n) = \sum_{k=1}^{K} P(x_n \in a_n^k) \cdot \mu_{A_j^n}(a_n^k) \tag{8}$$

$j=1,\ldots,|A_n|$, $n=1,\ldots,N$,

and an output variable

$$w(B_j) = P(B_j) = \sum_{k=1}^{K} P(y \in b^k) \cdot \mu_{B_j}(b^k) \tag{9}$$

$j=1,\ldots,|B|$.

Step 4: Choosing linguistic values from set C_1, that meet the requirement of minimum support (*min w*). Selected linguistic values will constitute set F_1 which consists of 1-element frequent sets.

$$\begin{aligned} F_1 = \{ & A_j^n \mid \forall j, n : w(A_j^n) \geq min\ w, \\ & B_j \mid \forall j : w(B_j) \geq min\ w \} \end{aligned} \tag{10}$$

Step 5: If F_1 set is not empty, shift to another point of algorithm assuming that $r=1$, as a number of linguistic values in a frequent set. Otherwise stop creating algorithm.

Step 6: Creating C_{r+1} set of $r+1$-element frequent sets candidates on the basis of F_r set, frequent r-element itemsets.

$$\begin{aligned} F_r = \{ & \{A_{j(1)}^{n(1)}, \ldots, A_{j(r-1)}^{n(r-1)}, A_{j(r)}^{n(r)}\}, \\ & \{A'^{n(1)}_{j(1)}, \ldots, A'^{n(r-1)}_{j(r-1)}, A'^{n(r)}_{j(r)}\}, \ldots\}, \end{aligned} \tag{11}$$

where some of linguistic values $A_{j(r-s)}^{n(r-s)}, s = 0,\ldots,r-1$ ($A'^{n(r-s)}_{j(r-s)}$) may constitute linguistic value $B_{j(r-s)}$ ($B'_{j(r-s)}$).

The set C_{r+1} consists of combination of elements of the set F_r meeting the condition

$$\begin{aligned} & (A_{j(1)}^{n(1)} = A'^{n(1)}_{j(1)}) \wedge (A_{j(2)}^{n(2)} = A'^{n(2)}_{j(2)}) \wedge \ldots \wedge \\ & (A_{j(r-1)}^{n(r-1)} = A'^{n(r-1)}_{j(r-1)}) \wedge (A_{j(r)}^{n(r)} \neq A'^{n(r)}_{j(r)}). \end{aligned} \tag{12}$$

Then the set C_{r+1} is equal

$$C_{r+1} = \{A_{j(1)}^{n(1)}, A_{j(2)}^{n(2)}, \ldots, A_{j(r-1)}^{n(r-1)}, A_{j(r)}^{n(r)}, A'^{n(r)}_{j(r)}\}. \tag{13}$$

Each element of the set constitutes a linguistic value belonging to a different input or output variable. Sets with recurrent elements must be deleted.

Step 7: Calculating support measure for $r+1$-element frequent sets candidates

$$\begin{aligned} & w(A_{j(1)}^{n(1)}, \ldots, A_{j(r+1)}^{n(r+1)}) = P(A_{j(1)}^{n(1)} \cap \ldots \cap A_{j(r+1)}^{n(r+1)}) = \\ & \sum_{k(1)=1}^{K} \cdots \sum_{k(r+1)=1}^{K} [p_{k(1)\ldots k(r+1)}(x_{n(1)}, \ldots, x_{n(r+1)}) \\ & T(\mu_{A_{j(1)}^{n(1)}}(a_{n(1)}^{k(1)}), \ldots, \mu_{A_{j(r+1)}^{n(r+1)}}(a_{n(r+1)}^{k(r+1)}))], \end{aligned} \tag{14}$$

where T stands for *t-norm* operator (e.g. min).

It needs to be noted that some of variables $x_{n(r-s)}, s = -1, 0, \ldots, r-1$ may constitute variable y. In such case $A_{j(r-s)}^{n(r-s)} = B_{j(r-s)}$ and $a_{n(r-s)}^{k(r-s)} = b^{k(r-s)}$.

Step 8: Selecting $r+1$-element frequent sets C_{r+1} candidates, which meet the assumption of a minimum support and creating set F_{r+1} consisting of $r+1$-element frequent sets.

$$F_{r+1} = \{\{A_{j(1)}^{n(1)}, \ldots, A_{j(r+1)}^{n(r+1)}\} \mid w(A_{j(1)}^{n(1)}, \ldots, A_{j(r+1)}^{n(r+1)}) \geq min\ w\} \tag{15}$$

Step 9: If F_{r+1} set is not empty, repeat steps 6-9 assuming that $r=r+1$, as a number of linguistic values in a frequent set. Otherwise move to step 10.

Step 10: Creating a probabilistic fuzzy model rule base on the basis of frequent sets F_{r+1}. When value of minimum support is high enough, then $r=N$ and, in such case, on the basis of step 8 one receives a set of elementary rules in the following form:

$$\begin{aligned} & (w(A_{j(1)}^{n(1)}, \ldots, A_{j(N)}^{n(N)}, B_{m/j})) \text{ IF } x_1 \text{ is } A_{j(1)}^{n(1)} \text{ AND...} \\ & \quad \text{AND } x_N \text{ is } A_{j(N)}^{n(N)} \\ & \quad \text{THEN } y \text{ is } B_{m/j}. \end{aligned} \tag{16}$$

The rule base in the form of (1) is received by combining elementary rules (16) of the same rule antecedent. Rule weights are calculated as follows:

$$w_j = \sum_{m=1}^{M} w_m, \quad w_{m/j} = \frac{w_m}{w_j}, \tag{17}$$

where $w_m = w(A_{j(1)}^{n(1)}, \ldots, A_{j(N)}^{n(N)}, B_{m/j})$ is the probability of occurrence of events in elementary rules with the same antecedent (x_1 is $A_{j(1)}^{n(1)}$ AND..AND x_N is $A_{j(N)}^{n(N)}$), M – number of rules with the same fuzzy events in the antecedent.

5. EXPERIMENTAL RESULTS

The presented algorithm has been tested on an example of "sombrero" datasets, which are generated via a two-parameter function described by the equation:

$$z = \sin\sqrt{x^2 + y^2} \Big/ \sqrt{x^2 + y^2}\,. \tag{18}$$

It was assumed that $x \in$ <-10,10>, $y \in$ <0,20>. Training sets (8963 triples) and testing sets (1037 triples) were randomly chosen.

Seven fuzzy sets were defined for each variable x and y from the model. An example of grades of membership for variable x belonging to fuzzy sets is presented in Fig. 2. Other grades of membership of variable y are defined analogously.

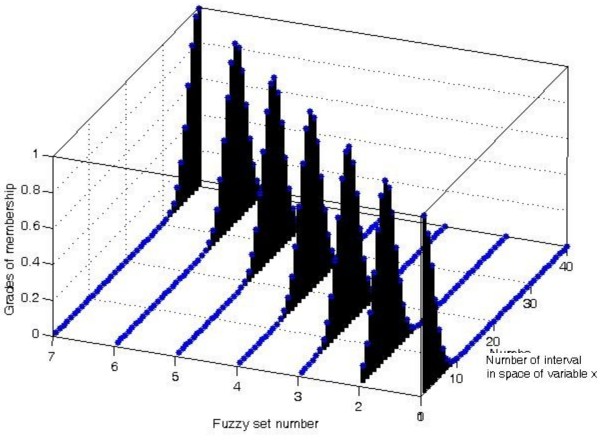

Fig.2. Grades of membership for variable *x*.

In order to define the influence of algorithm on the complexity of the model, time of generating system knowledge base and inference time as well as a number of elementary rules were tested depending on minimum support (*min w*). Results were also tested via analysis of errors RMSE (Root Mean Square Error) that occurred as a result of inference relying on the knowledge base with the use of training and testing sets.

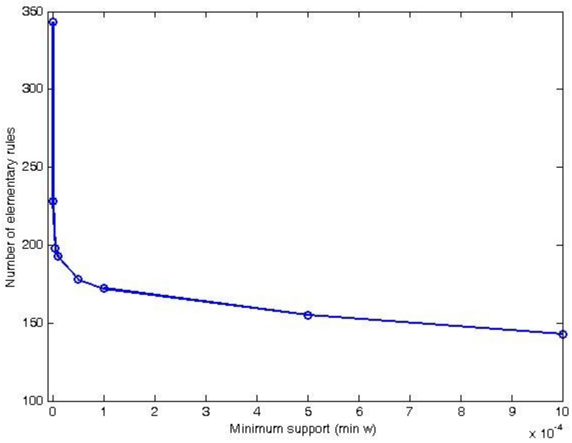

Fig.3. Number of elementary rules of a model depending on minimum support (*min w*).

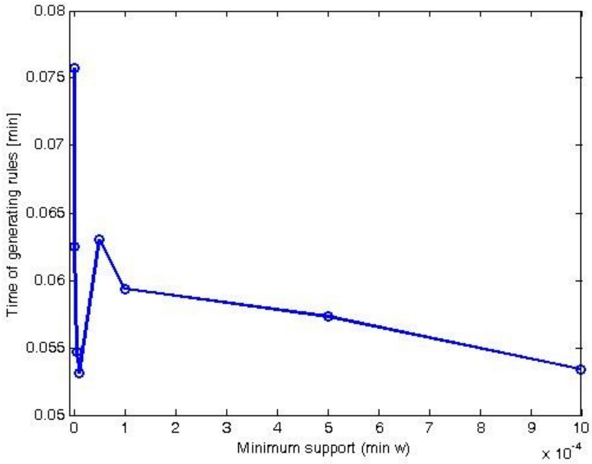

Fig.4. Time of generating knowledge base depending on minimum support (*min w*).

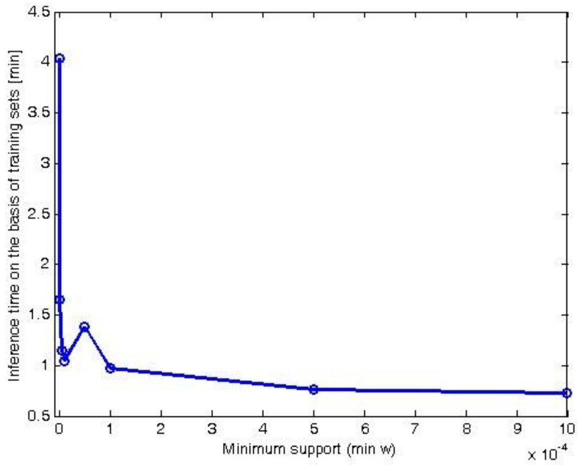

Fig.5. Inference time depending on generated knowledge base in relation to minimum support (*min w*).

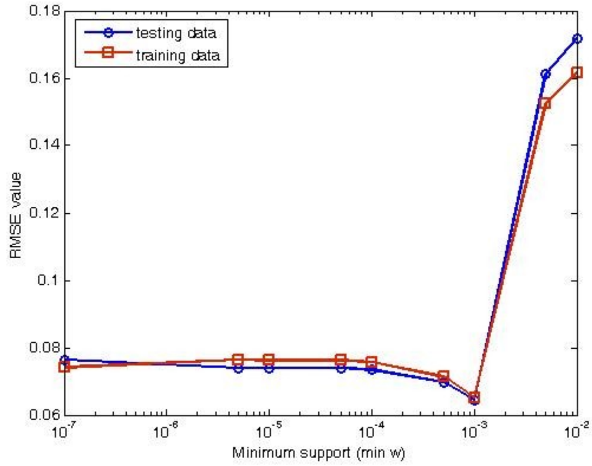

Fig.6. Value of RMSE errors for training and testing sets depending on minimum support (*min w*).

As shown in Fig. 3 the number of elementary rules is reduced as the value of minimum support increases. It causes significant reduction of inference time in relation to a model with created probabilistic-fuzzy knowledge base (Fig. 5) as well as a slight reduction of rule base generation time (Fig. 4). Despite the fact that complexity of the model is reduced, its capacity to recreate a function (18) is similar and even improves. Only surpassing a certain value of minimum support (*min w*>10^{-3}) causes significant worsening of the quality of the model.

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